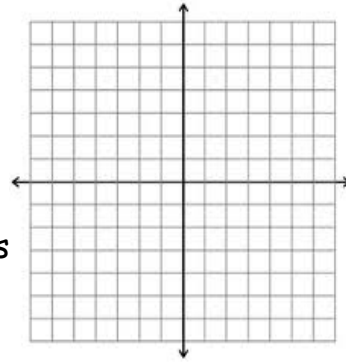


Precalc Warm Up # 13-4

$$\vec{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$$



- Find the angle between the vectors \vec{a} and \vec{b}
- Find a vector that is 5 long in the direction of \vec{b} .
- What is the magnitude of $\vec{a} - 2\vec{b}$?

EXERCISES 12.4 p. 427

1. If $a = i + 7j - k$ and $b = 4i + 7j + 5k$ find:

(i) $4a$

(iv) $2(a - b)$

2. The position vectors of A and B are $\vec{OA} = -3i + 4j - 2k$ and $\vec{OB} = i - 4j - 3k$. Find:

(i) \vec{AO}

(iv) $3\vec{OA} + 6\vec{BO}$

3. The vectors p and q are defined by $p = \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$ and $q = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}$. Find:

(i) $p + 2q$

(iv) $2p + 3q$

- $$A \begin{pmatrix} 7 \\ 7 \\ 4 \end{pmatrix} - 3 \begin{pmatrix} 3 \\ -1 \\ B \end{pmatrix} = \begin{pmatrix} -37 \\ -25 \\ 5 \end{pmatrix}$$
- $$7A - 9 = -37 \quad -4(4) - 3B = 5$$
- $$7A = -28 \quad \vdots$$
- $$A = -4 \quad \downarrow$$

9. Two vectors are defined as $a = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$ and $b = \begin{pmatrix} 6 \\ -6 \\ -5 \end{pmatrix}$. Find values of the scalars X and Y if $Xa + Yb$ is equal to:

(i) $\begin{pmatrix} -36 \\ 32 \\ 33 \end{pmatrix}$ (ii) $\begin{pmatrix} 30 \\ -22 \\ -31 \end{pmatrix}$ (iii) $\begin{pmatrix} -12 \\ 24 \\ 1 \end{pmatrix}$

i)

$$X \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix} + Y \begin{pmatrix} 6 \\ -6 \\ -5 \end{pmatrix} = \begin{pmatrix} -36 \\ 32 \\ 33 \end{pmatrix}$$

check
 $4x - 5y = 33$

$$-3x + 6y = -36$$

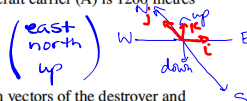
$$x - 6y = 32$$

$$-2x = -4$$

$x = 2 \rightarrow$ plug in & find y , then

10. A submarine (which is considered the origin of the vector system) is 60 metres below the surface of the sea when it detects two surface ships. A destroyer (D) is 600 metres to the East and 800 metres to the South of the submarine. An aircraft carrier (A) is 1200 metres to the West and 300 metres to the South.

(a) Define a suitable vector basis for this problem.



(b) Using the submarine as the origin, state the position vectors of the destroyer and the aircraft carrier.

(c) A helicopter pilot, based on the aircraft carrier wants to make a supplies delivery to the destroyer. Find, in vector terms, the course along which the pilot should fly.

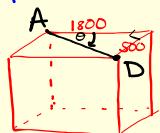
$\vec{SD} = \begin{pmatrix} 600 \\ -800 \\ 60 \end{pmatrix}$ $\vec{SA} = \begin{pmatrix} -1200 \\ -300 \\ 60 \end{pmatrix}$

e) $\vec{AD} = \vec{AS} + \vec{SD}$
 $= \begin{pmatrix} 1200 \\ 300 \\ -60 \end{pmatrix} + \begin{pmatrix} -600 \\ 800 \\ 60 \end{pmatrix}$

Book answer describes the path relative to the submarine. $= \begin{pmatrix} 1800 \\ -500 \\ 0 \end{pmatrix}$

more info:

$$|\vec{AD}| = \sqrt{1800^2 + (-500)^2 + 0^2} \approx 1868.2 \text{ metres}$$



$$\tan \theta = \frac{500}{1800}$$

$$\theta = 15.5^\circ$$

$$E 15.5^\circ S$$

EXERCISES 12.5

p. 431

1. Find the lengths of these vectors, expressing your answers as surds. It is not necessary to simplify these surds.

(i) $i + 3j$

(viii) $\begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$

2. Find unit vectors in the same directions as these vectors:

(i) $4i + 4j$

$$\sqrt{4^2 + 4^2}$$

$$\frac{1}{\sqrt{32}}(4i + 4j)$$

$$\frac{1}{4\sqrt{2}}(4i + 4j)$$

$$\frac{i + j}{\sqrt{2}}$$

(viii) $\begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}$

3. A mass sitting on the ground is being pulled by a force of 4 Newtons in a Northerly direction, 3 Newtons in a Westerly direction and 1 Newton upwards.

- (i) Express the forces acting on the mass in terms of an appropriate vector basis.
 (ii) Find the total magnitude of the force acting on the mass

4. (a) Find a vector of length 3 units in the direction of $i - j + k$.
 (b) Find a vector of length $\sqrt{3}$ units in the direction of $3i - j + \sqrt{2}k$.
5. The vectors $a = 2j + 4k$ and $b = xi + 3k$ are of equal length. Find x .

$$|\vec{a}| = \sqrt{4+16} = \sqrt{20} \quad |\vec{b}| = \sqrt{x^2+9}$$

$$(\sqrt{x^2+9})^2 = (\sqrt{20})^2$$

$$x^2 + 9 = 20$$

$$4a) \frac{3}{\sqrt{3}} (i - j + k)$$

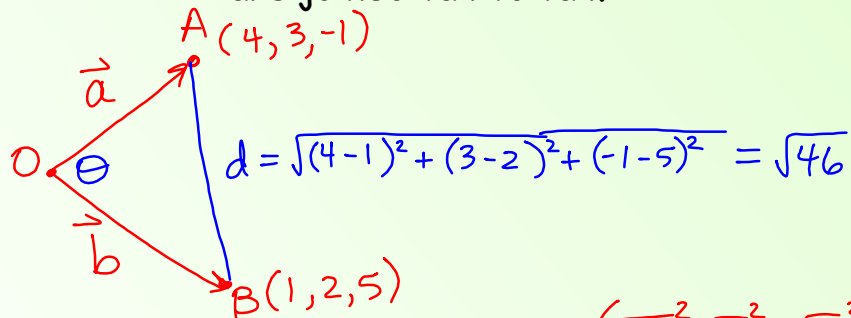
$$b) \frac{\sqrt{3}}{\sqrt{9+1+2}}$$

$$2\sqrt{3} \leftarrow \frac{\sqrt{3}}{\sqrt{12}} (3i - j + \sqrt{2}k)$$

$$\frac{1}{2} (3i - j + \sqrt{2}k)$$

Let $a = 4i + 3j - k$ and $b = i + 2j + 5k$
 Find θ , the angle between the vectors when they are joined tail to tail.

Let $\vec{a} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\vec{b} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$
 Find θ , the angle between the vectors when they are joined tail to tail.



$$|\vec{a}| = \sqrt{16+9+1} = \sqrt{26} \quad \theta = \cos^{-1} \left(\frac{\sqrt{26}^2 + \sqrt{30}^2 - \sqrt{46}^2}{2\sqrt{26}\sqrt{30}} \right)$$

$$|\vec{b}| = \sqrt{1+4+25} = \sqrt{30} \quad \theta \approx 79.7^\circ$$

Group Review.

HW: finish group review worksheet,
 due turned in tomorrow.